The Game of Same and Different — A Framework for Analyzing Determiner Systems[1]

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Abstract. This paper proposes to extend the *Game of Same and Different* (introduced by Grønn and Szæbø, 2012), and to transform it into a general framework for reasoning about determiner systems in natural languages. The system is applied to a Latin-like language with a singular-plural system and no articles. It will be shown that the system converges under highly rational agents toward the use of the demonstrative as a definite article, and against the emergence of an indefinite article. The behavior of agents more limited in their rationality is explored, and compared with the diachronic development of definite and indefinite articles. Finally, the underlying assumptions of the model are checked against two Latin texts.

Keywords: Determiner Systems, Definite Article, Indefinite Article, Rational Speech Act Model.

1. Introduction

One core idea of structuralism in linguistics is that linguistic items form *systems*, where the behavior and distribution of a linguistic expression are not only determined by its intrinsic meaning, but also by the meanings of other items it competes with (see Jakobson, 1971 for an application to verbal categories). While the intuition itself has come back into favor in recent years (see, e.g. Sauerland, 2008b), as far as I am aware, there has been no attempt as complete as the one by Jakobson in order to account for a reasonably complex, and possibly full, linguistic subsystem. One of the reasons is the wealth of interacting constraints, which are difficult to reason through verbally with any degree of confidence. This paper proposes to overcome these difficulties by providing a framework for determiner systems in natural languages, using a Rational Speech Act (henceforth RSA) solution concept[2] (see, e.g., Franke, 2017).

The idea that determiners form a system is not a new one. For instance, it has been known at least since Hawkins (1991) that the availability of a *definite* article in a given grammatical context has an impact on the meaning effects triggered by the *indefinite* article in that context, as is illustrated in (1) vs. (2) (see Grønn and Szæbø, 2012; Amsili and Beyssade, 2016).

(1) I met a wife of John.
$\rightsquigarrow$ John has/had multiple wives

(2) John has a wife.
$\not\rightsquigarrow$ John has multiple wives

(3) a. I met the wife of John/John's wife.

[1]I would like to thank the reviewers and the audience at Sinn und Bedeutung in Osnabrück for their feedback and criticism. Furthermore, I would like to thank Lieven Danckaert, Michael Franke, and Igor Yanovich. Without their help and discussion, this article would either not have been written at all, or it would look very different.

Finally, my gratitude goes to the two *Hiwis* who proofread this paper, and who pointed out several errors. All remaining typos and errors are mine alone.

All counts, diagrams and data manipulation have been performed with Python 3.

[2]The pragmatic part of the game could be implemented with minor changes in an Iterated Quantal Response framework, although not in a Iterated Best Response framework (see, e.g., Franke and Jäger, 2014).

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b. #John has the wife.

For instance, in some contexts – as exemplified in (1) – an indefinite article triggers a non-uniqueness inference. However, as is shown by (2), this inference is not systematically triggered by the English indefinite article. The difference is explained by the fact that in the grammatical context of (1), the definite article could be felicitously used (see (3a)), whereas this is not the case for (2), since (3b) cannot mean that John is married to a unique woman.

A similar phenomenon comes from the semantics of plurals (see, e.g., Link, 2000). Intuitively, the use of a plural seems to indicate that we are faced not with one, but with several instantiations of a predicate (see (4)):

(4) a. John owns a house.
b. John owns houses.

However, there are good reasons to assume that plurals in languages like English do not exclude singletons from their denotation (a view known as "inclusive plural"):

(5) Do you have children?
a. Yes, one.
b. #No, one.

Assuming that the speaker has exactly one child, the appropriate answer to (5) has to be (5a), and cannot be (5b). If the meaning of the plural excluded the atoms (here: individual children), one would expect the truthful answer to be (5b), and not (5a). This once again points to the idea that in many instances a plural is infelicitous because there is a more specific alternative (namely the singular), and that there is no *semantic* impossibility of it being used in contexts like (4), in order to denote that John owns a single house.

If we take serious the ideas that determiner systems form systems, we will have to deal with at least these two phenomena, and possibly several others more. As the numbers of competing constraints increase, reasoning them through verbally becomes more and more difficult,[3] and therefore, a formal account would be preferable.

A formal account of systems might also be profitable for two further domains of linguistic investigation, namely diachrony and typology. We know, for instance, that unity cardinals like *one* are one of the diachronic sources of indefinite articles (see, e.g., Heine, 1997; Dryer, 2013b; Kuteva et al., 2019), and also, that demonstrative determiners are one of the diachronic sources of definite articles (see, e.g. Dryer, 2013a; Kuteva et al., 2019). However, as far as I am aware, there has been little investigation into the idea that the presence of one of these items might accelerate or slow down the diachronic change of the other. This, however, might be a possibility, since, for instance in Romance, the definite article appears before the indefinite article (see, e.g., Carlier, 2001: 66f. for French) – which might be an accident, or not.

Similarly, according to the *World Atlas of Language Structures* (see Dryer, 2013a, b),[4] the largest group of languages has no articles at all, while those who have articles tend to have

[3]For instance, does the order of application of different constraints (e.g., plural vs. singular first, and then indefinite article vs. definite article) matter, or not? If the number of parameters is small enough, this can simply be tried out, but in case of multiple binary choices, we face a combinatorial explosion.

[4]Source: https://wals.info/combinations/37A_38A#2/25.5/148.5, consulted on 28/08/2019. Notice that these numbers diverge slightly from what is given in the individual chapters on definite and indefinite articles, namely https://wals.info/chapter/38 and https://wals.info/chapter/37, where the global numbers are higher, but the proportions are similar.

Type of language	Number
No articles at all	198
Both indefinite and definite article	154
Definite, but no indefinite article	89
Indefinite, but no definite article	40
Total	481

Table 1: Article Systems in the World's Languages according to *WALS*

both indefinite and indefinite articles. Among languages having only one type of article, those having only a definite article outnumber those having only an indefinite article roughly 2:1, as is illustrated in table 1.

Once again, the question is whether there is some kind of explainable pattern hidden in these numbers – and there might very well be none. But as of now, we lack the means to investigate such questions coming from diachrony and typology. The aim of this paper is to provide a tool, in order to start to investigate them. The main focus here will be on outlining a way on how to deal with determiner systems in the context of the game of same and different, rather than on confronting it with (diachronic or typological) data.

The remainder of the paper is structured as follows: section 2 introduces the game of same and different, describing both the states and the forms involved. In section 3, this game will be analyzed using a Rational Speech Act model, both in its behavior in the limit, and in contexts of bounded rationality. Section 4 confronts the predictions and assumptions of the model with sample data from classical Latin. Section 5 concludes the paper.

2. The Game of Same and Different

The *Game of Same and Different* is a signaling game, with in the most general case *n* states of the world that a speaker signals, and for which *m* different messages can be used. The hearer then tries to map the sent message back to a state of the world. It was originally conceived by Grønn and Szæbø (2012) in order to deal with the choice of definite vs. indefinite articles, and the presence or absence of a novelty effect associated with the indefinite article.

2.1. The Game of Same and Different by Grønn & Szæbø

Grønn and Szæbø (2012) observe that very often, the use of an indefinite article is associated with a novelty effect, as is illustrated in (6).

(6) Richard heard the Beaux-Arts Trio last night and afterwards had a beer . . .
 a. . . . with the pianist.
 b. . . . with a pianist.

The use of the indefinite article in (6b) suggests that Richard had a beer with a pianist who is not one of the members of the Beaux-Arts Trio, whereas the use of the definite article in (6a) precisely suggests that the pianist is the pianist who is a member of the Beaux-Arts Trio. This meaning effect can be seen as the result of a novelty effect associated with the indefinite article.

However, as Grønn and Szæbø (2012) note, this novelty effect is not present everywhere, otherwise, a sentence like (7a) should be fully felicitous under the assumption of a competent speaker, which it is not; though, the more explicit *another* is appropriate:

(7) Richard heard the Beaux-Arts Trio with its new cellist and its new violinist last night and afterwards had a beer with…
  a. …?a cellist.
  b. …another cellist

They argue that the availability of the *another* leads to this infelicity, and they develop a formal account of the distribution of the definite article, the indefinite article, and *another*. Their account is, however, restricted to competition between singular forms, and deals with the question of whether the referent of *determiner N* is identical to or different from an entity already established in discourse.

Before extending their idea, let us see how this is represented in the framework of a signaling game. We have two states that could be signaled, namely *identity* and *difference* (as formalized in (8)); and three forms at our disposal, namely *a N*, *the N*, and *another N* (as illustrated in (9)).[5]

(8)
  a. identity: $\frac{x|P(x)}{z|P(z), z = x}$
  b. difference: $\frac{x|P(x)}{z|P(z), z \neq x}$

(9)
  a. $a \mapsto \lambda P.\lambda Q.\frac{|}{z|P(z), Q(z)}$
  b. $the \mapsto \lambda P.\lambda Q.\frac{x|P(x)}{z|P(z), Q(z), z = x}$
  c. $another \mapsto \lambda P.\lambda Q.\frac{x|P(x)}{z|P(z), Q(z), z \neq x}$

Obviously, if we want to account for a full determiner system, this basic outline has to be extended, which we will do in section 2.2.

2.2. The Full Game of Same and Different

In order to separate the full framework (applicable in principle to any type of language and number system) from its application to some particular language, I will proceed by considering the very abstract general scheme, and then apply this scheme as an illustration to a language with a singular-plural system.

Generally, we will have to consider two elements: the *presupposition* and the *assertion*. I will

[5] The notation in the rest of the paper uses the flat DRT notation by Sauerland (2008a) (where the numerator corresponds to the presupposed content, and the denominator represents the asserted content). For further illustration, the flat (ia) corresponds to the (probably more familiar) box notation (ib):

(i) a. $x, y|P(x), Q(y)$ b.

x, y
$P(x)$ $Q(y)$

start with the presupposition.

The presupposition (and thus, the Common Ground, henceforth CG) could be empty, or there might be some presupposition with respect to a predicate *P*:

(10) a. $[\![\text{empty CG}]\!] = \frac{}{\bullet}$

b. $[\![\text{non-empty CG}]\!] = \frac{\mathbb{x}|P(\mathbb{x})}{\bullet}$

Here and in what follows, $\mathbb{x}$ represents a number-neutral discourse referent; $\bullet$ represents some formula.[6]

The assertion part is more complicated, since there are more possibilities to consider:

(11) a. $[\![\text{existence}]\!] = \frac{\bullet}{\mathbb{z}|P(\mathbb{z})}$

b. $[\![\text{identity}]\!] = \frac{\bullet}{\mathbb{z}|P(\mathbb{z}), \mathbb{z} = \mathbb{x}}$

c. $[\![\text{difference}]\!] = \frac{\bullet}{\mathbb{z}|P(\mathbb{z}), \mathbb{z} \neq \mathbb{x}}$

d. $[\![\text{partitive}]\!] = \frac{\bullet}{\mathbb{z}|P(\mathbb{z}), \mathbb{z} \sqsubset \mathbb{x}}$

e. $[\![\text{superset}]\!] = \frac{\bullet}{\mathbb{z}|P(\mathbb{z}), \mathbb{x} \sqsubset \mathbb{z}}$

While *identity* and *difference* are already present in Grønn and Szæbø (2012), the present article also considers partial identity between the presupposition and the assertion, namely *partitive* and *superset*, and also considers cases where there is a pure assertion of *existence* (since an assertion can concern an entity not present in the common ground).

With these basic ingredients in place, we can now consider its application to some language, and combine the different types of presuppositions with the different types of assertion, given the background of the number system of the language.

2.2.1. States in the Full Game of Same and Different

Let us assume a language with a singular and a plural. If we specify the number-neutral variables by attributing singular or plural values, we obtain thus 3 different types of presupposition, namely an empty common ground, or a singular or a plural common ground. For the assertion, we have singular and plural varieties of each of the five basic types given in (11), giving us a total of 10 different types of assertion. Theoretically, thus, we could obtain $3 \times 10 = 30$ different combinations in a singular-plural language. In a language with an additional dual, we would obtain $4 \times 15 = 60$ different possibilities.

However, all possibilities are not consistent, and some are redundant. Consider for instance an empty common ground: In this case, we cannot have an identity relation with the assertion, nor a partitive. Similarly, for the identity relation between presupposition and assertion, the number of the presupposition and the assertion need to be identical. Therefore, in a singular-plural language, there are 12 different configurations that are neither redundant nor contradictory:

[6] I will dispense in what follows with λ notation for semantic representations.

(12) a. $\text{ENS} = \frac{|}{x|P(x)}$
b. $\text{ENP} = \frac{|}{X|P(X)}$
c. $\text{SNS} = \frac{z|P(z)}{x|P(x), x \neq z}$
d. $\text{SNP} = \frac{z|P(z)}{X|P(X), z \notin X}$
e. $\text{PNS} = \frac{Z|P(Z)}{X|P(X), x \notin Z}$
f. $\text{PNP} = \frac{Z|P(Z)}{X|P(X), \forall z \in Z, \forall x \in X : x \neq z}$
g. $\text{SIS} = \frac{z|P(z)}{x|P(x), x = z}$
h. $\text{PIP} = \frac{Z|P(Z)}{X|P(X), X = Z}$
i. $\text{PPS} = \frac{Z|P(Z)}{x|P(x), x \in Z}$
j. $\text{PPP} = \frac{Z|P(Z)}{X|P(X), X \subset Z}$
k. $\text{SSP} = \frac{z|P(z)}{X|P(X), z \in X}$
l. $\text{PSP} = \frac{Z|P(Z)}{X|P(X), Z \subset X}$

The three-letter descriptive labels in (12) read as follows: the first letter always refers to the common ground, which can be either *E*mpty, *S*ingular, or *P*lural. The second letter refers to the relation of the assertion with respect to the common ground, and may be *N*ew, *I*dentical, *P*art-of, or *S*uperset-of. The third letter refers to the assertion part, and may be either *S*ingular or *P*lural.[7]

To rehearse this, let us consider a few examples. For instance, *ENS* refers to a configuration where the common ground is empty with respect to the predicate, where there is a singular asserted, and where that singular is new with respect to the common ground. This corresponds to a typical context which would require an indefinite article in a language like English, as illustrated in (13a). *PIP* refers to a configuration where there is a plural entity in the common ground, and where the assertion concerns a plural entity which is identical to the element in the common ground. This corresponds to a context in which one would use either a demonstrative or a definite article in English (see (13b)). Then, let us consider the *PPS* case: here, there is a plural in the common ground, the assertion is a singular, and this singular is a part of the plural antecedent. In English, such a configuration seems to require the cardinal *one*, as is illustrated in (13c). Finally, consider an SNS configuration, where a plural in the common ground, and the asserted singular is new with respect to this plural. In English, *another N* would be used in such a configuration.

(13) a. One upon a time, there was *a donkey*. [ENS configuration]
b. We saw some donkeys and a few horses. *The donkeys* were looking at us. [PIP configuration]
c. We saw some donkeys and a few horses. *One donkey* was looking at us. [PPS configuration]
d. We saw a donkey in the bushes. *Another donkey* was standing under a tree. [SNS configuration]

Now that we have considered the states to be signaled,[8] we can move on to the forms.

[7] I assume that there are no empty assertions.

[8] The present version of the game of same and different includes every relation between the content of the presupposition and the content of the assertion. Notice, however, that we do not necessarily need to add the superset relation as a primitive to the system, since this could be derived compositionally as a combination of an *identity* statement (that is, either SIS or PIP) with one of the *new* statements (that is SNS, SNP, PNS and PNP), which would give us a more fine-grained taxonomy:

2.2.2. Forms in the Game of Same and Different

Whereas the states only depended on the number system of the language, the forms to be considered will depend on the language (which is a priori rather easy), and also on the question what is to be taken to be a relevant alternative in the determiner system of the language (which is a more difficult consideration, to which we will have to come back to in section 4).

For the sake of the argument, I will use a language with a singular-plural system (assuming an inclusive plural), assuming that language to have no 'real' articles, but nevertheless having a demonstrative, an expression signifying *other*, and a unity cardinal *one*. The semantics of these forms is given in (14). The aim is to be close enough to actually existing Latin in order to be able to discuss a choice of issues, without being bogged down by the difficulties (which will be discussed later in section 4).

(14)
a. $[\![\text{bare SG}]\!] = \frac{|\;|}{z|P(z),Q(z)}$

b. $[\![\text{dem SG}]\!] = \frac{x|P(x)}{z|P(z),Q(z),z=x}$

c. $[\![\text{one SG}]\!] = \frac{|}{z|P(z),Q(z),|P\cap Q|=1}$

d. $[\![\text{other SG}]\!] = \frac{\mathbb{x}|P(\mathbb{x})}{z|P(z),Q(z),z\neq\mathbb{x}}$

e. $[\![\text{bare PL}]\!] = \frac{|}{Z|P(Z),Q(Z)}$

f. $[\![\text{dem PL}]\!] = \frac{X|P(X)}{Z|P(Z),Q(Z),Z=X}$

g. $[\![\text{sev PL}]\!] = \frac{|}{Z|P(Z),Q(Z),|P\cap Q|\geq n}$

h. $[\![\text{other PL}]\!] = \frac{\mathbb{x}|P(\mathbb{x})}{Z|P(Z),Q(Z),Z\neq\mathbb{x}}$

At least the meaning for demonstrative is too simplified – while it certainly has to refer to an entity already in the common ground, there should be some additional accessibility condition to it (see, e.g., Acton, 2014); the meaning as it stands corresponds more or less to what is assumed to be a semantics for a definite determiner. We will come back to this issue in section 4 below.

2.3. Form-Meaning Matrix of the Game

Now that we have made assumptions about the states that a hearer wants to signal, and the linguistic forms to be used to that end, we can fill in a form-meaning matrix to our game. This is done as follows: for each form, we check whether it is compatible with the constraints of the situation. If it is compatible, we enter a 1 in the matrix (meaning thus 'grammatical'); if it is incompatible, we enter a 0 in the matrix (which means thus 'ungrammatical'). The result is

(i)
a. This donkey and (also) another donkey were looking at us. [SIS + SNS = SSP]
b. This donkey and (also) other donkeys were looking at us. [SIS + SNP = SSP]
c. These donkeys and (also) another donkey were looking at us. [PIP + PNS = PSP]
d. These donkeys and (also) other donkeys were looking at us. [PIP + PNP = PSP]

Since the superset configurations are of a more peripheral interest in this paper, I will tentatively maintain them in the set of states to be signaled.

	bare SG	dem SG	one SG	other SG	bare PL	dem PL	sev PL	other PL
ENS	1	0	1	0	1	0	1	0
ENP	0	0	0	0	1	0	1	0
SNS	1	0	1	1	1	0	1	1
SNP	0	0	0	0	1	0	1	1
PNS	1	0	1	1	1	0	1	1
PNP	0	0	0	0	1	0	1	1
SIS	1	1	1	0	1	1	1	0
PIP	0	0	0	0	1	1	1	0
PPS	1	0	1	0	1	0	1	0
PPP	0	0	0	0	1	0	1	0
SSP	0	0	0	0	1	0	1	0
PSP	0	0	0	0	1	0	1	0

Table 2: The Form-Meaning Matrix of the Full Game of Same and Different in Widest Scope Contexts

displayed in table 2.[9]

It is important to note that table 2 only represents grammatical contexts where the nominal element has widest scope; in configurations where the element appears under the semantic scope of another operator, the data might be different. For instance, if the determiners take scope below a universal quantifier, the bare singular will no longer be semantically equivalent to the demonstrative singular, since the demonstrative requires a wide-scope interpretation with respect to the universal quantifier, which the bare form does not.[10] Therefore, the table represents the acceptability of a given form with respect to a given meaning *in a given grammatical context*.[11]

Table 2 incorporates the idea of an inclusive plural, where the bare plural form could in principle be used in all configurations. Notice also that the bare singular and "one singular", while not having the same meaning, can be used equivalently with respect to the game of same and different: where one is acceptable, the other is acceptable as well; where one is impossible, the other is impossible as well (the same is true for the bare plural vs. "several plural").

Furthermore, notice that the demonstrative (in the singular as well as in the plural) is extremely specific, since it is only felicitous with one (for the singular: SIS) or two states (for the plural:

[9]Table 2 makes a somewhat generous assumption on the cardinality restriction of *several PL*; I have assumed that it will be appropriate if $n \geq 1$, in order to preserve the general symmetry between singular and plural, while assuming an inclusive plural. Nothing particular hinges on this assumption; the pragmatic outcome in the limit would be the same if *several PL* excluded singular instances.

[10]This fact can be illustrated for English with the opposition between the indefinite article and a demonstrative, as illustrated in (i). While the indefinite article shows scopal ambiguity (the familiar specific vs. non-specific distinction), this is not the case for the demonstrative.

(i) a. Every student saw a movie. [✓every student > a movie; ✓a movie > every student]
b. Every student saw this movie. [*every student > this movie; ✓this movie > every student]

[11]The reasoning in Grønn and Szæbø (2012) crucially hinges on the fact that in narrow-scope contexts of *the*, *a* and *another*, their respective distribution is no longer the same as in widest-scope contexts.

SIS and PIP). The felicitous use cases for the demonstrative are a strict subset for the grammatical contexts for the bare singular and plural, respectively. The pattern of the distribution of a form being a strict subset of another also holds for the relation between the bare forms (singular and plural) and '*other*' (singular and plural): whenever the use of *other* is grammatical, the use of a bare form would also be grammatical; the inverse, however, is not true.

3. The Game of Same in Different in RSA

In order to obtain a behavioral prediction from table 2, we need to apply some kind of pragmatic framework to it. I will use here the Rational Speech Act Model (see, e.g., Goodman and Stuhlmüller, 2013; Franke, 2017; Scontras et al., 2018), even though related models from the iterated response family (though not iterated best response) would yield comparable predictions.

The basic idea in such a framework is that a minimally pragmatic speaker (or Speaker_1) will choose the form to signal anticipating the reaction of a literal hearer. Similarly, a minimally pragmatic hearer (or Hearer_1) will anticipate the behavior of the minimally pragmatic speaker in order to maximize the success of the communication. More generally, a speaker_n will anticipate a hearer_{n-1}, and a hearer_n will anticipate a speaker_n. The procedure can be stated as in Franke (2017):

(15) a. $\text{Speaker}_{n+1} = \frac{\exp(\lambda * \text{EU}_S(m,t,\text{Hearer}_n))}{\sum_{m'} \exp(\lambda * \text{EU}_S(m',t,\text{Hearer}_n)))}$

b. $\text{Hearer}_{n+1} = \frac{Pr(t) \times \text{Speaker}_{n+1}(m|t)}{\sum_{t'} Pr(t') \times \text{Speaker}_{n+1}(m|t')}$

where m, m' = linguistic forms sent by the speaker, t, t' = states of the world, and λ is the softmax-parameter, indicating on how rational the agents will perform.

The Expected Utility of a speaker, EU_S is defined as follows:

(16) $\text{EU}_S(m,t,\text{Hearer}_n) = \log(\text{Hearer}_n(t|m)) + \text{Hearer}_n(t|m) \times c_m$

This version of the RSA thus assumes that speakers have preferences with respect to the cost of the message they send (the [inverse] cost of a form is expressed in (16) by c_m), whereas hearers have preferences with respect to the frequency of the states (see the weighting with the probabilities of the states t in (15b)).

In the setup of the game, I assumed that speakers prefer shorter forms to longer forms, and fewer words to more words. It is quite obvious that the bare forms should be less costly than messages with a demonstrative or *other*; however, it is less clear if we should generally assume singulars to be less costly than plurals. It turns out that for our purposes, it does not matter if singulars are more or less costly than plurals (at least, as long as the difference is located in a reasonably close range of values); the one thing that does matter for the outcome is that *one SG* be more costly than the bare *SG*, and that *several PL* be more costly than the bare plural[12] — which do not seem to be very problematic assumptions to make.

Concerning states, table 3 assumes that situations where the item is new or identical are more

[12]The reason for this is that most effects on the distributions of the messages in the systems are technically scalar implicatures, which are not sensitive to message weight in RSA. However, we will observe a specialization effect between the bare forms and the cardinal forms, which is technically a manner implicature, and this does depend on differential speaker and hearer preferences.

frequent than situations with partitive and superset configurations. This particular assumption (or more specifically, that the new configuration is more frequent than the partitive configuration) is required, and we will see in section 4 below that this is indeed the case. I have also assumed that singular states are more frequent than plural states, but nothing specific hinges on this assumption — at least, as far as the convergence behavior of the system is concerned.[13]

In what follows, I will assume that the speaker knows precisely the state of the world, and that there are no errors in picking a message. Furthermore, I assume that the hearer has to infer the state based on only the message and a prior on the states, but that there is no further contextual information helping with that inference. Both of these assumptions are questionable, but the one made on the hearer is probably more consequential, and less of an innocent idealization. The issue is the following: a state like SIS has two components: the presupposition and the assertion. The presupposition concerns items that are in the common ground, and it is a not entirely unreasonable assumption that the common ground is indeed common, and shared between speaker and hearer.[14] There are certainly cases where an element in the common ground has to be accommodated, but this is not the rule. As it stands, the model assumes that all elements in the common ground have to be accommodated.[15]

3.1. Behavior in the Limit — Convergence

If given sufficient leeway to converge (for instance, by choosing a highly sophisticated speaker of level 10, and a softmax-parameter of 5), this will produce the prediction of behavior as indicated in table 3.

Thus, the RSA model predicts that the use of the demonstrative should become obligatory (which is a scalar implicature), for whatever precise meaning it has (and the assumed meaning above is clearly not specific enough, since it lacks some kind of accessibility condition from the common ground), and provided, it is an alternative in the first place.

The use of the unity cardinal is predicted to specialize for PPS-situations (which is a manner implicature), and this is driven by the differential in cost for the speaker, and the differential in

[13]Once again, this is due to the fact the RSA does not require differential frequencies of states for cases of scalar implicatures, but only for manner implicatures.

[14]This problem was pointed out to me by Michael Franke (p.c.).

[15]I have tried to quantify the extent of the problem using two Latin texts in PROIEL, namely Caesar's *Gallic Wars* and Cicero's *Letters to Atticus*, as illustrated in the table below. As expected, the proportion of accommodated DPs varies widely following the text type and assumptions an author/speaker can make with respect to the reader/hearer: Cicero's letters had a specific addressee, with whom the author shared many background assumptions; Caesar's history of the Gallic Wars was aimed at the general Roman public of the time.

	Caesar (GW)	Cicero (LA)
# anaphoric	1034	156
# accommodated	379	157
% accommodated	26.8	50.2

Technically, this count has been done as follows: among the list of nouns annotated for information structure in the text, the anaphoric category counts the nouns annotated as "old" or "old_inact" (that is, nouns that have a textual antecedent); the accommodated category counts nouns annotated as "acc_sit" (that is, accessible in the situation) or "acc_inf" (accessible by inference). For a description of these labels, please refer to Haug et al. (2014). This way of counting is unlikely to be the best way to quantify accommodation, and should not be seen as anything better or more than a very rough approximation.

	bare SG	dem SG	one SG	other SG	bare PL	dem PL	sev PL	other PL
ENS	1	0	0	0	0	0	0	0
ENP	0	0	0	0	1	0	0	0
SNS	0	0	0	1	0	0	0	0
SNP	0	0	0	0	0	0	0	1
PNS	0	0	0	1	0	0	0	0
PNP	0	0	0	0	0	0	0	1
SIS	0	1	0	0	0	0	0	0
PIP	0	0	0	0	0	1	0	0
PPS	0	0	1	0	0	0	0	0
PPP	0	0	0	0	0	0	1	0
SSP	0	0	0	0	0	0	1	0
PSP	0	0	0	0	0	0	1	0

Table 3: Convergence of the Game of Same and Different for Speaker_{10}, with $\lambda = 5$

frequency for the hearer (partitive situations being rarer).

Furthermore, the RSA-model accounts for the exclusive use of the plural (which is once again a scalar implicature).

Therefore, if one considers these results from the point of view of grammaticalization, rational use is enough to explain (at least parts of) the grammaticalization of definite articles, since the model predicts it to be used exclusively in cases of identity once it has been established as an alternative in the game of same and different. Indeed, the basic obstacle the demonstrative has to face in order to become a definite article is restrictions on its meaning.

However, rational use as embodied in the present RSA model should block (or at least, slow down) the grammaticalization of a unity cardinal into an indefinite article, since the bare singular is involved in a manner implicature which specializes its use for ENS contexts, and where the unity cardinal is restricted to PPS contexts. Therefore, the grammaticalization of indefinite articles must be due to other factors, which have been tentatively identified in Schaden (to appear) as *emphasis* (see also Ahern and Clark, 2017 for an application of a similar idea to Jespersen's cycle).

3.2. Bounded Rationality and the Emergence of Articles

The results in table 3 were about what the system would eventually converge to (given rather sophisticated speakers and hearers, or also, given enough time). However, actual humans are probably not (all) as rational and sophisticated when having to face the pressure of communication in real time.

One of the advantages of the RSA model is that it allows us to investigate what happens if we relax the assumptions of high rationality, and take as a base more realistic speakers and hearers (that is, speakers and hearers that are more constrained by real time pressure, and less sophisticated).

In RSA, there are two proxies for rationality and sophistication: First, the depth of counterfac-

	bare SG	dem SG	one SG	other SG	bare PL	dem PL	several PL	other PL
ENS	0.50349	0.00000	0.44655	0.00000	0.02560	0.00000	0.02435	0.00000
ENP	0.00000	0.00000	0.00000	0.00000	0.51249	0.00000	0.48750	0.00000
SNS	0.04041	0.00000	0.03584	0.85231	0.00205	0.00000	0.00195	0.06742
SNP	0.00000	0.00000	0.00000	0.00000	0.02876	0.00000	0.02736	0.94387
PNS	0.04041	0.00000	0.03584	0.85231	0.00205	0.00000	0.00195	0.06742
PNP	0.00000	0.00000	0.00000	0.00000	0.02876	0.00000	0.02736	0.94387
SIS	0.00127	0.96180	0.00112	0.00000	0.00006	0.03567	0.00006	0.00000
PIP	0.00000	0.00000	0.00000	0.00000	0.00180	0.99647	0.00171	0.00000
PPS	0.50349	0.00000	0.44655	0.00000	0.02560	0.00000	0.02435	0.00000
PPP	0.00000	0.00000	0.00000	0.00000	0.51249	0.00000	0.48750	0.00000
SSP	0.00000	0.00000	0.00000	0.00000	0.51249	0.00000	0.48750	0.00000
PSP	0.00000	0.00000	0.00000	0.00000	0.51249	0.00000	0.48750	0.00000

Table 4: Behavior of Determiner System given the Simplest Pragmatic Speaker (S_1), with λ=3

tual reasoning: do I merely assume a minimally pragmatic speaker (or hearer), who reacts to a literal hearer (or speaker), or should we assume a more sophisticated pragmatic speaker of level n, who reacts to a pragmatic hearer of level $n-1$? In section 3.1, I assumed a speaker of level 10, whereas in psycholinguistics, generally, only the minimally pragmatic speaker of level 1 is assumed. Second, there is the issue of how sensitive a speaker (or a hearer) should be when confronted with small differences, which is expressed with the softmax parameter λ. If λ is set to 0, a speaker will choose a form uniformly at random; with very high values for λ, a speaker will choose nearly deterministically the form best corresponding to his preferences, even though a second form may be only slightly dispreferred.

Assuming the simplest pragmatic speaker (S_1), and $\lambda = 3$, we obtain the prediction in table 4. Here, we no longer obtain categorial predictions (that is, in this context, only use form x), but probabilities of use. There are two striking facts that deserve discussion in table 4: first, even with a relatively unsophisticated speaker, the demonstrative singular is predicted to be used in 96% of the instances of SIS contexts, and its main competitor in this context is the demonstrative plural with 3.5%. In the PIP context, the demonstrative plural is predicted to appear in over 99% of cases.

This strong picture of specialization for identity contexts already at low degrees of sophistication strongly contrasts with the low degree of specialization we obtain for ENS and ENP contexts. Here, the bare forms are predicted to appear very slightly more frequently than 50%, and the unity cardinal (or the *several* form) are not far behind (at 44% and 48%, respectively). This is no artefact of the settings for λ and the degree of recursion employed for table 4; we can plot the predicted difference between the probability of the marked form (that is, the demonstrative or the unity cardinal) and the probability of the unmarked form (that is, the bare form) for the SIS and ENS contexts, respectively, for different values of λ and different depths of recursion, as is illustrated in figure 1.

In table 4, if a line hits either 1 or -1, that means that the system has converged — either toward the marked form, if the line is at 1, or to the unmarked form, if the line is at -1. The reason is the following: 1 or -1 can only be reached if one of the two forms has reached a probability of 1, and the other has a probability of 0. Each line represents a degree of recursion (where the

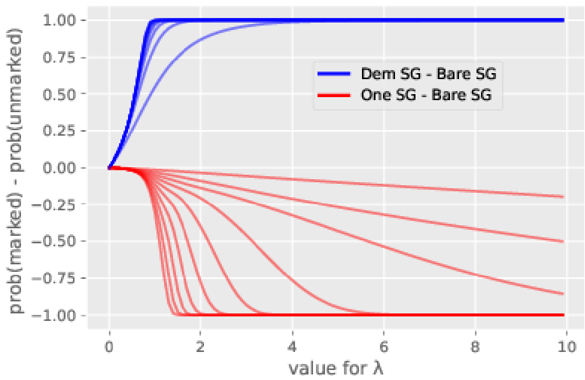

Figure 1: Difference between Marked and Unmarked Form for Different Levels of Sophistication

line closest to the center represents S_1, and lines further out speakers S_2 to S_{10}, with λ going from 0 to 10, as we move on the x-axis. As can be seen, the convergence in the ENS context to the bare form is *much* slower than the convergence toward the demonstrative in the SIS context.

So, let us sum up the result of this figure: the less rational a speaker, the more instances of *one* there will be for the ENS scenario, but the less rational a speaker, the fewer instances of the demonstrative there will be for the SIS scenario. However, the speed of convergence for the demonstrative outpaces by far the speed of convergence against the unity cardinal. Maybe this difference in speed gives the unity cardinal a fighting chance to be pushed by other factors, but it is clearly to early to draw firm conclusions.

One potential problem we should be aware of at this point is that the difference in speed might be an artefact of the (RSA) model used here, and not reflect any cognitively real difference between the two processes. The issue is the following: RSA models predict a fast convergence in cases of scalar implicature, whereas the derivation of manner implicatures is a slower and more involved process (see figure 4). At this point, it is not certain that we can simply assume that this is a reflection of an underlying greater cognitive cost of processing manner implicatures vs. scalar implicatures. It would be possible that in an RSA model, any tentative of fitting the speed of a diachronic development that is based on a scalar implicature vastly overestimates the speed of a diachronic development based on a manner implicature. In the end, this is an empirical question, which cannot be answered in the current paper.

However, I would like to stress that, while the difference in *speed* might thus be an artefact, the difference in *direction* is certainly not, and suggests a major difference in the diachronic evolution of definite and indefinite articles.

4. A Brief Look at (Classical) Latin

Up to now, my basic concern was to present the outline of the system, and to determine its predictions. To this end, I have considered a language that was basically a sanitized version of Latin. But what about real Latin? Do the assumptions made to derive the predictions hold up to reality? I will briefly consider some of the necessities in order to arrive at the predictions.

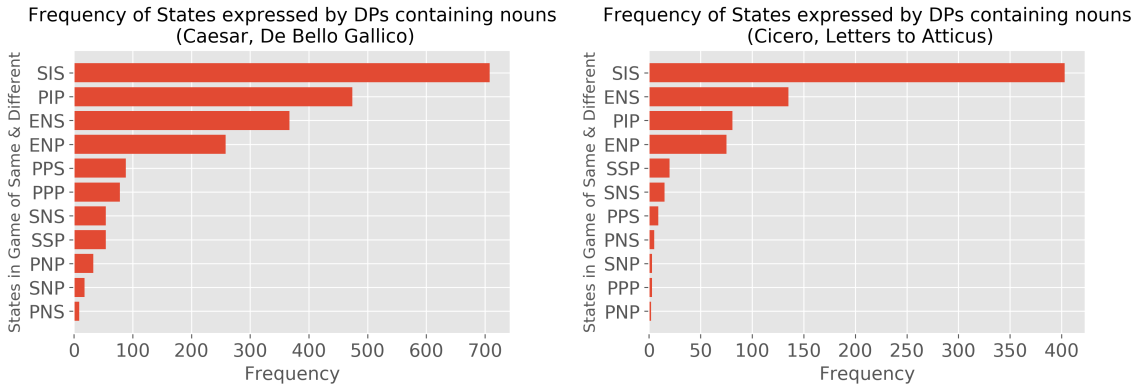

Figure 2: States of the Game of Same and Different in Caesar (left) and Cicero (right)

Let me briefly recapitulate the basic facts we would like to derive: the Latin demonstrative *ille* eventually became the definite article in most current Romance languages, and the Latin unity cardinal *unus* eventually became the indefinite article in current Romance languages.

In order to evaluate the predictions we get from the RSA model, we need a corpus annotated syntactically, and also annotated for the game of same and different. Such a corpus does not exist, but we can derive such a corpus from the annotations in PROIEL (see Haug and Jøndal, 2008; Haug et al., 2014), which provides us with 3 Latin texts that have been fully annotated for information structure.[16] This gives us a very limited number of texts, and whose annotation has to be transformed, but it gives us a useful starting point to check some of the assumptions.

In order to derive the model, a crucial ingredient was the assumption that partitive contexts (notably PPS and PPP) are less frequent than *new* contexts (namely ENS and ENP); the general question underlying this is the distribution of the states of the game, as far as NPs (or DPs) are concerned. As it turns out, the distributions vary slightly, but in all cases, SIS is the most frequent state, with ENS second or third, and PPS and PPP further down the line. The assumption of the model is thus validated. The general distributions can be illustrated for Caesar and Cicero in figure 2, which shows a count of NPs/DPs which contain a noun (be it common a common or a proper noun). This count thus excludes strictly pronominal anaphora.

While both of these texts have been written in the same year, by male members of the Roman senatorial aristocracy, they belong to rather different *genres* (political commentary or historiography vs. personal letter), which may in part influence the distribution of the states.

Let us now come to the predictions with respect to the forms. The model predicts that the demonstrative should be used for all instances of the SIS and PIP states. This clearly is an incorrect prediction, since the immense majority of cases of these states, bare forms are used (as is illustrated in table 5 for Caesar's *Gallic Wars*;[17] the same pattern also appears in Cicero).

So, what are the reasons for this divergence between the predictions of the model, and actually occurring classical Latin? First, the model assumed that there is *one* demonstrative determiner

[16]The fully annotated texts are Caesar's *Gallic Wars*, Cicero's *Letters to Atticus*, and the *Pilgrimage of Egeria*. The gospels from Saint Jerome's *Vulgate* are partially annotated.

[17]Notice the presence of a NA column in this table. This means that the annotators of the PROIEL corpus were not able to attribute an information status to the noun.

	Bare Sing	HIC Sing	IPSE Sing	IS Sing	UNUS Sing	ALT Sing	AL Sing	Bare Plur	HIC Plur	IPSE Plur	IS Plur	MULT Plur	AL Plur
ENS	350	3	0	11	8	1	2	0	0	0	0	0	0
ENP	0	0	0	0	0	0	0	252	0	1	4	1	1
SNS	52	0	0	1	2	2	1	0	0	0	0	0	0
SNP	0	0	0	0	0	0	0	18	0	0	1	1	0
PNS	8	0	0	1	1	0	0	0	0	0	0	0	0
PNP	0	0	0	0	0	0	0	31	0	0	0	1	1
SIS	635	16	3	53	1	0	0	0	0	0	0	0	0
PIP	0	0	0	0	0	0	0	450	15	1	8	0	0
PPS	77	0	0	3	2	0	0	0	0	0	0	0	0
PPP	0	0	0	0	0	0	0	76	1	0	0	0	0
SSP	0	0	0	0	0	0	0	49	0	0	1	2	0
NA	584	3	1	14	7	0	3	231	2	0	3	3	0

Table 5: DP forms and States in the Game of Same and Different in Caesar, *Gallic Wars*

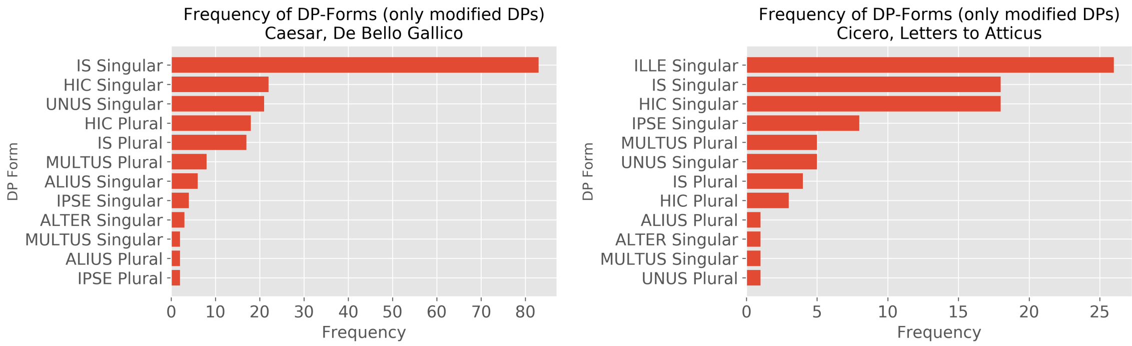

Figure 3: Frequency of Adnominal Determiners in Caesar vs. Cicero

in the language. However, classical Latin (just like contemporary English) has several of those, namely *hic, ille, ipse* and *is*, and there is no single obvious candidate for the game of same and different. The same is true for *other*: instead of having one form to express such a meaning, Latin has two forms, namely *alter* and *alius*.

The best explanation is probably that — at least in the mid-first century BC — there was no demonstrative present as an alternative in the game of same and different. One possible argument for this is the difference in the uses of the adnominal determiners in Caesar and Cicero. While *ille* is the most frequent demonstrative in adnominal use in Cicero, it does not appear once in adnominal position in Caesar's *Gallic Wars*. This is illustrated in figure 3. The point is that we would not expect a strong divergence in the frequencies of use if these were really strongly grammatically constrained items. Another possible argument against introducing some demonstrative as an alternative into the game of same and different is that its semantics does not necessarily fit. The problem is the following: while a demonstrative definitely requires its referent to be localizable in the common ground, it normally comes with further accessibility conditions (for instance, on distance with respect to the speaker; see, e.g., Acton, 2014: 63), especially when there are several demonstratives in a language.

And if we remove the demonstrative forms (singular and plural) from the form-meaning matrix,

	bare SG	one SG	other SG	bare PL	several PL	other PL
ENS	1	0	0	0	0	0
ENP	0	0	0	1	0	0
SNS	0	0	1	0	0	0
SNP	0	0	0	0	0	1
PNS	0	0	1	0	0	0
PNP	0	0	0	0	0	1
SIS	1	0	0	0	0	0
PIP	0	0	0	1	0	0
PPS	0	1	0	0	0	0
PPP	0	0	0	0	1	0
SSP	0	0	0	0	1	0
PSP	0	0	0	0	1	0

Table 6: Game of Same and Different without Demonstratives (Speaker$_{10}$, with $\lambda = 5$)

the predicted outcome of convergence is that the bare singular is used deterministically for both ENS and SIS contexts, and also that the bare plural is used deterministically for both ENP and PIP contexts, as is shown in table 6.

Similarly, the question is whether we should assume that *alter* and *alius* are a part of the game, since the states of SNS, SNP, and PNP are more often expressed by the bare forms than by the marked forms with *alius* or *alter*; or whether we should assume that *unus* or *multus* (the equivalent of '*several*') are alternatives at this time, since *unus* fails to make a dent into the share of the bare singular in the PPS context, and *multus* is not used in PPP contexts.

If we further remove the unity cardinal and the equivalent of *several* as forms from the game of same and different, the prediction is that the bare singular is used deterministically for PPS contexts, and the bare plural for PPP contexts. Further removing *other* from the forms will result in a situation where the bare singular is used for all singular states, and the bare plural for all plural states. This seems to be the best fit prediction for table 5.

This, however, raises the question how we could determine *a priori* and in a principled way which kind of message has to be considered at a given time in a given language, and what causes some form to become an alternative in the game. It seems clear in any case that one will not be able to rely on frequency alone in order to determine this fact, since *(an)other* in English has a rather low frequency, and is still required as an alternative for the game of same and different (as has been argued, convincingly, by Grønn and Szæbø, 2012). Another criterion could be semantic: a form is an alternative in the game of same and different if its semantics allows for it, and it has to represent exactly one or more of the states in example (12).

Summing up, the basic assumptions on the frequencies of states seem to be sound (if we can extrapolate from these two Latin texts), while the crucial issues (which are much harder to control) seem to be i) what criteria determine the availability of a given form as an alternative in the game of same and different; and ii) what processes could cause a form that was previously not an alternative in the game to become an alternative.

5. Conclusions and Perspectives

In this paper, I have tried to show that (in principle arbitrarily complex) linguistic systems can be modeled in an RSA model (or other versions of iterated response models), and the specific example chosen to illustrate this were determiner systems, based on the game of same and different, generalized from Grønn and Szæbø (2012). Such models can be of great use for investigating multiple, interacting — and possibly conflicting — constraints.

One obvious application for such a model is the investigation of the diachronic development of determiner systems. Potentially, an RSA model can provide quantified predictions of frequencies, which can be checked against texts in corpora. In the present paper, I have only provided a basic check against the distribution of (proto-)determiners in Latin, based on the Latin texts in the PROIEL corpus. The main problem of deploying the game of same and difference on a larger scale, and to be of use in tracking the diachronic development of determiner systems is that this requires corpora that are annotated for both syntax and information structure, and in order to be fully comparable, ideally in a format that is compatible with PROIEL. As far as I am aware, such corpora do not exist as of now, and even for Latin, the number of texts in PROIEL is very small (as compared to the number of surviving texts). Since reasoning on systems requires an estimation of many different parameters (speaker preferences with respect to every form; hearer assumptions with respect to the frequencies of every state), and that there seems to be considerable variation among existing sources (see figures 2 and 3), we would ideally like to have as many texts as possible.

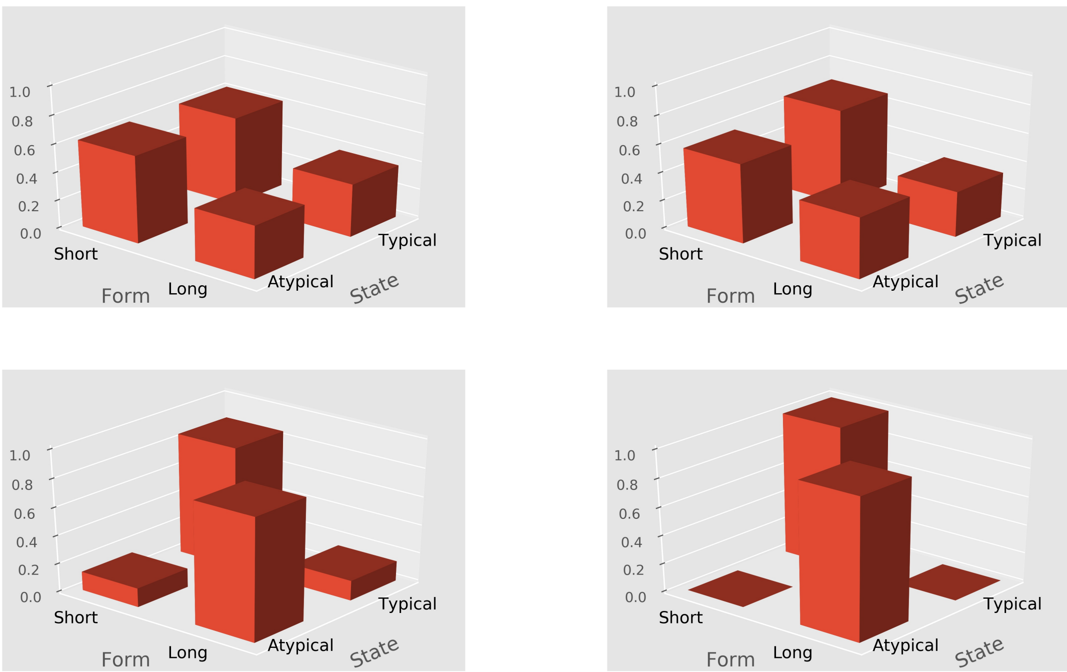

Figure 4: Convergence of Manner Implicature for Speaker$_1$ (top left) to Speaker$_4$ (bottom right)

There is possibly one technical (or theoretical) obstacle which might limit the applicability of RSA models (and also of Iterated Quantal Response models) in their current form as formalized in Franke (2017), and this concerns the differing resource requirements these frameworks predict for scalar vs. manner implicatures. Scalar implicatures in these frameworks converge with considerably less sophisticated agents than manner implicatures (as is illustrated in figure 4, a manner implicature only really starts converging with $Speaker_3$; a scalar implicature already converges more strongly with $Speaker_1$). While comparative rates of change do not matter as long as we only consider the behavior in the limit, it might turn out to be important if we want to predict language change. If manner implicatures are really cognitively more demanding than scalar implicatures, they should also take longer to show an effect in diachrony. However, I am not aware of any research into the comparative cognitive load associated with different types of implicatures, and the differential might turn out to be an artefact of this type of iterated response frameworks, rather than a fact about language processing.

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